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Magnetic fields of stars: using stars as tools for understanding the origins of cosmic magnetic fields

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Phil. Trans. R. Soc. Lond. A 2000 **358**, 689-709
doi: 10.1098/rsta.2000.0553

THE ROYAL SOCIETY

Magnetic fields of stars: using stars as tools for understanding the origins of cosmic magnetic fields

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I survey the status of research on the detection and quantitative measurement of stellar magnetic fields, and discuss theoretical ideas which try to account for the origins of these magnetic fields, consistent with present observations.

Keywords: stars; magnetic fields; magnetic dynamo; stellar activity; stellar rotation

1. Introduction

Certain effects of magnetic fields on stars are readily apparent even to the naked eye, and historical records of such effects—sunspots—are found among the written records of our most ancient civilizations. The demonstration that sunspots are in fact related to magnetic fields is, however, a modern achievement, and occurred just over 90 years ago, when Hale (1908) took advantage of the then just-discovered Zeeman effect to show that the light from sunspots yields the characteristic line splitting associated with strong magnetic fields, while the surrounding solar photosphere does not. Then, as now, one asks: why is the Sun magnetized? Then, as now, one answers: we cannot as yet give a complete first-principles account of the origin of solar magnetic fields. This apparent lack of progress is, however, enormously deceptive, because we now know much about the phenomenology of solar magnetic fields; and by using observations of stars similar to the Sun, we have learned much about the relationship between stellar properties and the properties of stellar magnetic fields. Theory has also made enormous strides, especially in the past two decades, and is now reaching levels of sophistication that allow theory to make productive contact with observations. It is my aim in this paper to provide a status report of these advances.

Before turning to the main subject matter of this paper, it is useful to point out that the observational and theoretical issues confronted in the stellar context find their counterparts in other astrophysical domains, such as planetary interior physics and studies of the interstellar medium in our galaxy as well as in other galaxies. The common element is the problem of explaining the presence of strong magnetic fields, which turn out to be ubiquitous in astrophysical systems. The advantages offered by stars are the exquisite detail with which the spatial and temporal structure of solar magnetic fields can now be studied, and the broad range of stellar properties that can be explored by observing magnetic fields on other (solar-like) stars. Thus, stars allow us to 'turn the knobs' on stellar magnetic dynamo theory, and to observe the consequences.

My plan is first to discuss how stellar magnetic fields are detected and measured, then to ask what types of stars turn out to be 'magnetic' and to discuss the properties of magnetic fields on such stars. Finally, I will turn to the question of why we might expect stars to be magnetic objects in the first place. I will primarily concentrate on stars similar to our Sun, because understanding such stars is most likely to be fundamental to understanding magnetism of most other types of stars.

2. How do we know that stars are magnetized?

Following Hale's discovery of solar magnetic fields, astronomers were naturally led to ask whether the Sun was unique in this respect—a possibility that few if any scientists thought was very likely. However, demonstrating the Sun's non-uniqueness proved to be very difficult, primarily because Hale's technique could not be directly adapted to detect Zeeman splitting of lines from other stars. Success did not come for almost half a century, when Babcock (1947; see also Preston 1967) managed to detect strong magnetic fields in a star—78 Virginis—the first star other than the Sun whose field was directly detected. Hundreds of similar stars were subsequently found, but these stars were distinctly non-solar, most with spectral types ranging from late B to early A on the main sequence; Babcock's star, for example, turned out to be a classic example of an Ap star.

(*a*) *How are stellar magnetic fields detected?*

Babcock's method (based on directly measuring the splitting of spectral lines of photospheric origin) is an example of a *direct* probe for stellar magnetic fields: such probes are unambiguous signatures of the presence of stellar magnetic fields, in the sense that alternative explanations for the observed effect either do not exist, or are implausible. Direct probes include the following.

Resolved Zeeman effect. This spectroscopic probe relies upon measurement of the splitting of the two σ components of a magnetically sensitive line:

$$\Delta\lambda \sim B \cdot \lambda^2 g_{\text{eff}}, \tag{2.1}$$

where λ is the wavelength of the unsplit line, B is the line-of-sight magnetic-field strength and g_{eff} is the effective Landé g factor. Typically, one chooses photospheric lines with large Landé factors and lying in the infrared (to maximize the sensitivity); examples include the 1.50 μm line of Fe I (with $g_{\text{eff}} = 3$) and the 2.231 μm line of TiO (with $g_{\text{eff}} = 2.5$).

Unresolved Zeeman effect. One might hope to extend the Zeeman technique to stars with relatively weaker line-of-sight magnetic fields by measuring the broadening in excess of thermal broadening of lines such as the above Fe I and TiO lines; this method's simple-minded implementation fails because processes other than the Zeeman effect can lead to line broadening in stellar photospheres. The ingenious trick pioneered by Robinson (1980; Basri *et al.* 1992; Saar 1996*a*, *b*) is to observe 'unmagnetized' lines (i.e. lines associated with very small or zero Landé g factors) formed in a similar temperature domain as the magnetically sensitive lines, and to use the observed line broadening of these lines as a measure of the non-magnetic contributions to the broadening of magnetically sensitive lines.

Spectropolarimetry. Since the σ components are (circularly) polarized, there is the possibility of observing net polarization in the wings of magnetically sensitive lines. This technique has seen successful application; for a review, see Donati 1999), but also suffers from sensitivity limitations because of the need to accurately measure signals with relatively low intrinsic polarization at high dispersion. There has also been some exploration of detecting broad-band linear polarization by integrating over the magnetically sensitive line profile (a method that relies upon the differential saturation of the σ and π Zeeman components); this method is even more challenging in its application. In any case, polarimetric observations of a point source—as are all stars other than the Sun—must rely upon a net effect when integrating over the visible stellar surface. Spatially resolved solar observations, however, suggest that stellar surface fields are likely to be extremely inhomogeneous, on scales ranging from the stellar radius down to the scale of energy-containing eddies in the star's photosphere; we therefore expect substantial cancellation, and thence little if any detectable residual polarization. It should therefore not be too surprising that spectropolarimetry has not been particularly successful as a field-detecting technique for solar-like stars. In the case of rapidly rotating stars, there remains the possibility that large-scale flux concentrations can be detected if different polarities segregate in longitude; this type of Zeeman–Doppler imaging has been particularly explored by the St Andrews group (see, for example, Brown *et al*. 1991; Jardine *et al*. 1999).

Gyromagnetic effects. Gyromagnetic effects—typically synchrotron or cyclotron emission (and polarization) due to energetic electrons travelling in the magnetized outer atmospheres of stars, and observed at radio wavelengths—also provide direct evidence for the presence of stellar magnetic fields. Radio observations are limited to those portions of a star's atmosphere which are optically thin in the radio, i.e. to those portions of a star for which the local plasma frequency lies below the radio frequency of observation. Consequently, radio observations largely probe stellar magnetic fields in the coronae of stars, while the Zeeman measurements probe for the presence of magnetic fields at photospheric levels of stars. Until fairly recently, limited sensitivity meant that radio observations indicating the presence of stellar magnetic fields were largely associated with stellar flares (Davis *et al*. 1978; Kahler *et al*. 1982). However, more sensitive studies have shown considerable evidence for steady radio emission from active stars (such as the dMe stars), especially below 15 GHz (Güdel & Benz 1989); this emission is presumably associated with fast electrons gyrating in the strong (coronal) magnetic fields thought to dominate the outer atmospheres of these stars.

The direct methods can be used for only a relatively small fraction of stars observable by space and ground-based telescopes, since they typically require both relatively strong magnetic fields (so that the magnetic effects can be easily distinguished in the presence of significant sources of noise) and high sensitivity (because spectral dispersion leads to reduced signal-to-noise ratios). As a consequence, only the brightest stars, with relatively strong mean magnetic fields, can be examined. Indeed, the Sun, when viewed as a star, would appear to be an unmagnetized object if observed with one of these direct observational methods. In order to deal with the more common solar-type stars, or with stars which are far removed from us, a number of *indirect* probes for stellar magnetic fields have been developed; these probes cannot give a completely unambiguous signal for the presence of stellar magnetic fields, but rather depend upon a key assumption, namely that the phenomenology associated with solar magnetic fields finds its counterparts (albeit at times in the extreme) in stars.

Stellar surface inhomogeneities. The most straightforward indirect method for detecting stellar magnetic fields relies upon the idea that active stars should show evidence for the stellar analogue of sun spots. Such starspots—which in the solar analogy are assumed to be cooler than the surrounding non-magnetic photosphere—are in principle detectable by searching for photometric light-curve variations whose period matches the rotational period of the underlying star (Oskanyan *et al*. 1977). As long as the lifetime of such surface inhomogeneities is substantially longer than the stellar rotational period, it is now known that a combination of multicolour photometry with detailed modelling of limb darkening allows non-trivial reconstruction of the surface distribution of the cool starspot material (see, for example, Harmon 1999). In addition, photometry of tracers whose temperature sensitivity is such that they signal the presence of cool starspot material (O'Neal *et al*. 1996), as well as the measurement of the temporal variation of Doppler shifts of such tracers during the course of stellar rotation ('Doppler imaging'; Vogt & Penrod 1983), have been used as further constraints on light-curve inversions. These methods thus provide geometric information about photospheric magnetic fields, but no quantitative information about the magnetic-field strengths.

Activity. In a similar vein, it is assumed that the solar analogy also applies to observations of stars in the radio, ultraviolet and extreme ultraviolet, as well as at X-ray wavelengths. As a consequence, stellar observation of the Ca II H and K emission, as well as of extreme ultraviolet and X-ray emissions, is commonly attributed to the presence of stellar magnetic fields. Strictly speaking, the analogy itself is not particularly persuasive since alternative methods of producing the observed radiations which do not rely upon stellar magnetic fields are easily constructed. What is persuasive is instead the observation that the mean level of these radiations is closely correlated with the rotation period of the underlying stars (Pallavicini *et al*. 1981); no explanation other than one based on the solar analogy for stellar activity has been able to account for this correlation. This perspective on stellar activity actually substantially predates space-based observations of stars: in the case of quiescent emission, it had long been argued that stellar Ca II H and K emission should be regarded as analogues of the corresponding solar phenomenon (see, for example, Eberhard & Schwarzschild 1913; Wilson 1978); and in the case of optical (Kunkel 1970) and radio (Kahler *et al*. 1982) transients observed from the very late-type main-sequence stars referred to as flare stars, it has been commonplace to regard these transients as extreme counterparts of the solar-flare phenomenon (see van den Oord 1999). As in the case of starspots, the use of the activity analogy provides a probe for the presence of stellar magnetic fields, but does not lead to any quantitative estimates of their expected strength. Furthermore, attempts have been made to use temporal information (from either rotational modulation acting on long-lasting coronal or chromospheric structures, or from eclipsing systems) to deduce spatial structure of the emitting plasmas (Schmitt & Kürster 1993).

Plasma confinement. The presence of stellar magnetic fields can also be deduced by considering consequences of the mechanical stresses imparted by such fields. As

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an illustration, consider the observation of X-ray emission from a low-gravity giant star, such as Capella (Holt *et al*. 1979). X-ray photometry shows that the X-ray emission levels from Capella are roughly constant on a time-scale of a day or so; low-resolution X-ray spectroscopy shows that the plasma temperature of this emitting gas is rather high: as is the case for many active stars, the observed spectrum can be well fitted by two temperature components, the first at roughly $T_1 \sim 0.5\,\text{keV}$, and the other with T_2 well in excess of 1 keV. A simple calculation then shows that the plasma temperature for at least the higher-temperature component is above the escape temperature for this star. As a consequence, simple hydrodynamic models for Capella's atmosphere predict sudden evacuation of Capella's corona, inconsistent with the relatively steady level of observed X-ray emission.† The implication is that some other (non-gravitational) force must act to constrain Capella's corona from escaping; the obvious candidate is a magnetic field.

Stellar evolution and spin-down. The final indirect probe for stellar magnetic fields rests on the observation (Kraft 1967) that the rotational properties of stars on the main sequence show a highly systematic pattern: (low mass) main-sequence stars with outer stellar convection zones have systematically lower rotation rates than (high mass) stars whose outer envelopes are stable against thermal convection; the break between these two distinct main-sequence populations happens to occur right at the boundary on the main-sequence separating stars with convectively unstable outer envelopes from those with stable outer envelopes (e.g. early A spectral type). The simplest explanation for this observation again appeals to the solar analogy: stars with convective outer envelopes are thought to possess a magnetic dynamo; such a dynamo produces stellar magnetic fields which, upon their ejection from the stellar interior and coupling to wind outflows from the underlying star, lead to effective stellar spin-down. This argument is the least direct of all: strictly speaking, it argues only for the existence of stellar magnetic fields within the class of slowly rotating late-spectral-type main-sequence stars, rather than for the existence of magnetic fields associated with any particular, specific star.

(*b*) *What types of stars are magnetic?*

Using the methods just described, it has been possible to demonstrate that virtually all known types of stars are magnetic objects: late-type main-sequence stars, giants and supergiants, OB stars, T Tauri stars, white dwarfs and neutron stars. With the important caveat that 'absence of evidence is not evidence of absence', the only general class of stars for which *no* positive (direct or indirect) detections of magnetic fields exist is the early A stars (such as Vega and Sirius A). Indeed, even for the other class of stars which shows little if any evidence for activity (e.g. chromospheric or coronal emissions, or non-thermal radio emission), namely the very-late-spectral-type giants and supergiants, there is nevertheless substantial evidence for the presence of magnetic fields (Chapman & Cohen 1986; Szymczak & Cohen 1997).

† This evacuation is quite unlike a stellar wind. As first shown by Parker (1955), stellar winds arise in 'open' atmospheres whose temperature is typically of order of, but *smaller* than, the stellar escape temperature.

(c) *Properties of stellar magnetic fields*

The following discussion of stellar magnetic-field properties is based on the results obtained from the variety of direct and indirect observational techniques just discussed. In this context, it is essential to keep in mind that significant selection effects must be present: every one of the methods we discussed is subject to one or more selection effects which can skew our interpretation of what stellar magnetic fields are really like. This summary is specifically structured so as to make contact with the following theoretical discussions.

(i) *Ubiquity*

Stellar magnetic fields are ubiquitous. Only a remarkably small subclass of stars (namely, early main-sequence A stars) shows no evidence for the presence of magnetic fields. Theory must account both for the overall ubiquity of stellar magnetic fields, as well as for the lack of detection of such fields among the early dA stars.

(ii) *Field strengths, filling factors and geometry*

The literature is full of results that quote averaged stellar magnetic-field strengths that are often comparable with the local photospheric equipartition strength, and with a wide range of photospheric filling factors (Saar 1996*a*, *b*). Since selection effects are surely at work here, we need to keep in mind that if these results can be depended upon, then such stars represent the most magnetically active variety. In the case of stellar activity studies based on emission indicators (such as, for example, stellar X-ray emission), there has been a concerted community-wide effort to disentangle selection effects by, first, choosing statistically complete samples of stars; and, second, using modern statistical techniques (in particular, 'survival analysis'; Babu & Feigelson 1996). To the best of my knowledge, such an approach has yet to be applied to measurements of stellar magnetic fields; and until this is done, it will be difficult to say much of any generality on this subject.

Quite aside from these questions of statistical analysis, there is the difficulty of interpreting the measurements themselves. Researchers such as Rüedi *et al*. (2000) have argued that the interpretation of magnetic-field measurements using the methods discussed in § 2 *a* is at the mercy of the quality of the modelling that is done—in other words, the passage from stellar measurements to derived quantities such as the mean magnetic field and the magnetic-field filling factor is not at all straightforward, and still subject to considerable discussion.

Despite these caveats, what one can conclude from the extant data are two key things. First, it is clear that there exist stars—the most active ones—for which the photospheric fields are, in the *mean*, comparable in strength with the local equipartition field (e.g. the magnetic-energy density is comparable with the photospheric kinetic-energy density). Second, there also exist stars for which the photospheric covering factor for strong magnetic-field regions ('spots') can be of order a third of the visible hemisphere. These are challenging constraints, because a theory which has the ambition to account for stellar magnetic fields in general, but has been originally designed to explain solar magnetic fields, will nevertheless need to account for these far more remarkable observations.

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Finally, a short comment regarding the spatial distribution of 'starspots'. There has been a long history of reportage of starspots located at the rotational pole(s) of stars; these interpretations were also long discounted because—if the data are photometric in nature, especially using only a single passband—it is readily shown that polar spots appear naturally as part of the normalization procedure for the inversion of photometric light curves (Harmon 1999, and references therein). However, by using multiple passband and Doppler imaging, both photometric methods are able to surmount this difficulty; and there are now quite definitive results showing the existence of polar spots (Barnes *et al*. 2000, and references therein). I will not touch on this subject any further because of space limitations, but I do note that such observations can be quite constraining for models of magnetic-flux eruption from the interiors of rotating stars; examples of relevant discussions can be found in Choudhouri & Gilman (1987), D'Silva & Choudhouri (1991), Schüssler & Solanki (1992), Schüssler *et al*. (1996), DeLuca *et al*. (1997), Barnes *et al*. (2000) and Lister *et al*. (2000).

(iii) *Activity–stellar-properties correlation*

A number of observers realized that as soon as a substantial population of stars had been surveyed for activity indicators, one would be in a position to answer the key question: what stellar attributes correlate with the observed level of activity? Hints that stellar rotation would be the key attribute were gleaned from the very earliest datasets (Vaiana *et al*. 1981), and indeed efforts to look for correlations based on the initially very modest sized datasets succeeded remarkably: thus, Pallavicini *et al*. (1981) were able to show that stellar rotation removed most of the variance in the X-ray luminosity distribution for late-type stars; and similar efforts succeeded using datasets focusing on activity indicators such as Ca II H and K emission (namely Noyes *et al*. (1984), who were the first to note the possibly important role played by the Rossby number $Ro \equiv P_{\mathrm{rot}}/\tau_{\mathrm{c}}$, where P_{rot} is the stellar rotation period and τ_{c} is the convective turnover time). These results were consistent with a picture originally espoused by Skumanich (1972), who showed that if one examines Ca emission from late-type dwarf stars in nearby open clusters, then one sees a clear decline of this emission with age; given that these stars also showed a decline in rotation rate with age (Kraft 1967), it was not surprising to find that the activity level declined with rotation rate. Since that time, the completion of surveys based on the Einstein Observatory data and on the more recent ROSAT data have placed this subject on a firm statistical footing: we now have not only statistically complete samples to choose from, we even have volume-limited samples of stars available for correlation studies (Schmitt *et al*. 1995; Hempelmann *et al*. 1995; Pallavicini 1996; Schmitt 1997). One finds the following.

(1) The level of stellar activity for late-type dwarf stars does not seem to be correlated with spectral type/stellar mass.

(2) The range of observed mean X-ray surface fluxes coincides well with the range of observed (local) surface fluxes on the Sun: the X-ray brightest stars have mean surfaces fluxes comparable with those of solar flares, while the X-ray dimmest stars have mean surface fluxes comparable with those of solar coronal holes.

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(3) The level of stellar activity is related to the spectral hardness of the emission: stars with brighter coronae tend to have more emission from gas at high ($T >$ 1 keV) temperatures; this is consistent with a picture in which increasing levels of stellar activity are tied to an increasing contribution to the emission from solar-like 'active regions'.

(4) Stellar activity levels are very well correlated with stellar rotation: rapid rotators are X-ray bright, while slow rotators are X-ray dim. One can improve the correlation somewhat by substituting the Rossby number for the rotation rate; however, since the Rossby number of a given star remains a derived quantity whose precise value cannot be at this point determined (one needs to know the turnover time of the convection zone, as well as the mean stellar rotation rate within the convection zone), it is my view that any such improvements are largely a consequence of the fact that use of the Rossby number introduces a further degree of freedom into the fitting problem.

(iv) *Activity–magnetic-field correlation*

The types of studies just discussed were a strong motivation to seek a direct connection between stellar activity levels (measured by, for example, the X-ray luminosity) and the level of magnetic activity (as measured by the mean magnetic field). This observational task is easy to formulate, but not trivial to carry out since the difficulty of measuring stellar magnetic fields restricts such measurements to a much smaller set of stars than has been observed by techniques focusing on activity emission levels. Among the first substantial surveys of this sort were those reported by Saar (1996a, b; see also Johns-Krull & Valenti 1996; Rüedi *et al*. 2000), with the result that activity levels indeed can be shown to scale with the mean surface magnetic-flux density fB (where f is the magnetic-field surface-filling factor). For example, Saar (1996a, b) shows that both the mean X-ray surface flux (F_{x}) and the mean C IV surface flux ($F_{\mathrm{C\,IV}}$) scale with fB:

$$F_{\mathrm{x}} \propto (fB)^{1.0}, \quad F_{\mathrm{C\,IV}} \propto (fB)^{0.8}. \tag{2.2}$$

Note, however, that—as pointed out above in a different context—the values of these exponents cannot as yet be taken entirely seriously since the statistical analyses of these datasets do not take censoring into account.

(v) *Cycle-period–rotation-period correlation*

The observational studies discussed so far attempt to relate measurements taken over a relatively short time (namely, stellar X-ray or UV/EUV fluxes) with quantities that change over stellar evolutionary time-scales (namely, stellar rotation rate, stellar bolometric luminosity). Quite aside from the fact that the former quantities are known to fluctuate substantially over the course of a single stellar rotation period (which can range roughly from days to a month), we also know (from the Sun) that activity-related emissions vary substantially over the course of an activity cycle. In order to understand the sort of correlations discussed above on time-scales comparable with an activity cycle, one needs to look at a controlled set of stars over time periods of order of or longer than an activity cycle period; this was the ambition of Wilson (1978), which has now been realized by his successors (Baliunas *et al*. 1995).

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The key step, pioneered by Wilson, was to use the amplitude of stellar calcium emission as a proxy for stellar surface activity levels; this method has the substantial advantage over space-based techniques that it is relatively inexpensive to implement and maintain over the necessary time spans (e.g. decades). These long-term 'synoptic' studies have led to a number of important results, the most relevant for our purposes being the following.

(1) If one fixes attention to a narrow spectral-type range on the main sequence, then the long-term secular behaviour of active stars shows a distinct dependence on rotation period P_{rot}: the most slowly rotating stars show little if any temporal variation; the modestly rotating stars (i.e. P_{rot} comparable with that of the Sun) show distinctive periodic variations in mean emission, which seem to be best interpreted as reflecting the expected variations in emission with a stellar activity cycle; and the rapidly rotating stars are dominated by chaotic (i.e. non-periodic) long-term time variations.

(2) There is a correlation between the ratio of the cycle period to the rotation period ($P_{\mathrm{cycle}}/P_{\mathrm{rot}}$) and the inverse rotation period ($1/P_{\mathrm{rot}}$) among stars showing clear evidence for activity cycles.

(3) One can also compare the above ratio ($P_{\mathrm{cycle}}/P_{\mathrm{rot}}$) with other stellar properties, such as the convection-zone depth, colour ($B - V$; Baliunas *et al*. 1995) and age and Rossby number *Ro* (Brandenburg *et al*. 1998); in each case, one finds a dependence of the period ratio on the chosen stellar parameter.

An absolutely key question is how these results are to be interpreted. It is certainly to be expected that any comprehensive theory for stellar magnetism ought to be able to account for these general correlations, even granted the complications introduced by the fact that some of the quantities being correlated (such as *Ro*) cannot be computed with any degree of precision. Thus, one expects these results to serve as constraints on theories. A deeper question is whether one can take these constraints seriously, given the relatively modest sophistication of the theories to be constrained. Thus, Tobias (1998) has argued that uncertainties in the details of the kind of magnetic dynamo models typically compared with these correlation studies are sufficiently great that these correlations cannot be thought of as serious model constraints. In addition, the impact of censoring (e.g. Babu & Feigelson 1996) on these types of correlation studies remains to be explored.

3. Theory

Now that we have established the observational fundamentals, we can turn to a discussion of theory. My aim is to provide sufficient background on the basics of magnetic dynamo theory so that some of the key current issues regarding comparisons of theory and observations can be sensibly discussed.

(*a*) *What is a magnetic dynamo?*

The key ingredient in any theory of stellar magnetic fields is a process that accounts for the continual (re)generation of the observed magnetic fields. To set the stage for the subsequent discussion, it is useful to define this process—the magnetic dynamo—a bit more precisely.

In the classical *kinematic problem*, one asks for the properties of the flow $\boldsymbol{u}$, with $\boldsymbol{u}$ specified *a priori*, such that the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} \quad \text{with } \nabla \cdot \boldsymbol{B} = 0 \tag{3.1}$$

is linearly unstable, i.e. such that its solutions

$$\boldsymbol{B}_i(\boldsymbol{r}, t) = \mathrm{Re}\{\hat{B}_i(\boldsymbol{r}) \exp(\gamma_i t)\}, \quad \{\gamma_i\} \text{ complex}, \tag{3.2}$$

have the property that

$$\mathrm{Re}(\gamma_i) > 0 \tag{3.3}$$

for at least one γ_i (η is the magnetic diffusivity). This is the perspective adopted in much of the astrophysical literature (Parker 1979). However, the above seemingly straightforward definition turns out to be quite subtle if one slightly rephrases the question: can one determine whether, for a given flow $\boldsymbol{u}$, equation (3.1) is in fact linearly unstable? For sufficiently simple systems, for which one can construct analytical solutions to equation (3.1), it is straightforward to answer this question; but for systems for which the solutions to (3.1) must be obtained numerically, it may not be possible to answer the question from a practical point of view. Thus, as pointed out by Weiss and co-workers (Cattaneo *et al*. 1991), flows for which the system (3.1) is very close to marginal stability have the property that the growth rates may be arbitrarily small, so that determining stability or instability may in practice be very difficult.

Now consider the more complex (but also far more astrophysically relevant) case of the nonlinear problem, in which one seeks solutions to the coupled set of equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} \quad \text{with } \nabla \cdot \boldsymbol{B} = 0, \tag{3.4}$$

$$\rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\nabla p + \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F}, \tag{3.5}$$

for a specified forcing function $\boldsymbol{F}$ (which might be imposed explicitly, or might be determined by imposing a heat flux sufficient to drive thermal convection in the domain of interest); here p is the usual gas pressure, ν is the viscosity and $\boldsymbol{J}$ is the current density. In this case, one does not expect indefinitely exponentiating solutions; instead, one would like to know for what types of forcing do equations (3.4), (3.5) have solutions for which the magnetic field $\boldsymbol{B}$ is maintained indefinitely (or, at least for times much longer than the Ohmic diffusion time for the largest-scale magnetic-field component in the system).

With these preliminaries, we can now discuss the types of questions asked by dynamo theorists; these questions can be placed in a hierarchy of increasing complexity and sophistication.

(1) For what type of flows $\boldsymbol{u}$ is equation (3.4) unstable to exponentially growing solutions? The answer to this question is the domain of *linear kinematic dynamo theory*.

(2) For what type of flows $\boldsymbol{u}$ is equation (3.4) unstable to exponentially growing solutions at large wavelengths λ (where λ is much larger than the velocity correlation length)? The answer to this question is the domain of *linear kinematic mean-field theory*; all dynamo models descendent from the original Parker (1955) dynamo fall into this category.

(3) For what type of forcing $\boldsymbol{F}$ do equations (3.4), (3.5) have exponentially growing solutions? The answer to this question is dealt with by weakly nonlinear dynamo theory, in which the interactions between the most rapidly growing solutions from linear theory are taken into account.

(4) Finally, for what types of forcing $\boldsymbol{F}$ do equations (3.4), (3.5) have finite-amplitude solutions that are maintained indefinitely? The answer to this question is dealt with in fully nonlinear calculations, i.e. within the context of direct numerical simulations (DNS) of the dynamo equations.

(*b*) *Type of dynamos*

In the previous subsection, we classified dynamo calculations according to the treatment accorded to the hydrodynamic flow, and according to the treatment of nonlinear terms in the governing equations. There is yet a third way of classifying dynamo models, which focuses on fundamental physical properties of the model(s), and which we now discuss.

For simplicity, consider once again the kinematic problem exemplified by equation (3.1) with $\boldsymbol{u}$ specified, together with the divergence-free constraint on $\boldsymbol{B}$. As we have already discussed, solutions of equation (3.1) have the generic form

$$\boldsymbol{B}_i(\boldsymbol{r},t) = \mathrm{Re}\{\hat{B}_i(\boldsymbol{r})\exp(\gamma_i t)\}, \quad \{\gamma_i\} \text{ complex}, \tag{3.6}$$

with instability resulting if

$$\mathrm{Re}(\gamma_i) > 0. \tag{3.7}$$

Assume for the moment that the system is actually unstable, i.e. that $\mathrm{Re}(\gamma_i) > 0$ for at least one value of i. We also define an intrinsic hydrodynamic time, t_0, as the ratio of the integral scale ℓ_0 to the velocity at the integral scale, u_0. In that case, we can now compare two time-scales, namely the ratio of the hydrodynamic time to the inverse growth rate for this instability.

Case 1. $\mathrm{Re}(\gamma_i)\cdot t_0 \to 0$ as $Re_{\mathrm{m}}(\equiv u_0\ell_0/\eta) \to \infty$.

In this case, field generation is fundamentally diffusive in nature, and the governing time-scale is the magnetic diffusion time. This case is usually referred to as the 'slow dynamo' process; the 'α–Ω' dynamo pioneered by Parker (1955) is of this type, and essentially all modern attempts to account for the large-scale field of stars such as the Sun have adopted Parker's paradigm (though the details vary considerably!).

Case 2. $\mathrm{Re}(\gamma_i)\cdot t_0 \to \text{const.} \neq 0$ as $Re_{\mathrm{m}} \to \infty$.

In this second case, diffusion becomes irrelevant, and the governing time-scale is the hydrodynamics time; this case is usually referred to as a 'fast dynamo'. The existence of such a dynamo was first recognized by Vainshtein & Zel'dovich (1972),

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who also developed what has been long viewed as the prototypical fast dynamo, the so-called 'stretch–twist–fold' dynamo. (This simple model entails an iterated three-step process: a closed field line is stretched, then twisted 180° (to obtain a figure-of-eight-like configuration), and finally folded back onto itself (so that one ends up with two parallel closed loops lying on top of one another).)

(c) *The simplest 'kinematic' linear dynamo theory*

As already stated, the ambition of magnetic dynamo theory is to account for the observational properties of stellar magnetic fields, and the relationships between magnetic and other properties of (late-type) stars. Practically all attempts to carry out this program since the 1950s have focused on what would seem to be a very special restrictive dynamo model; this model is the following.

(1) Kinematic: the properties of the flow $\boldsymbol{u}$ are specified (more precisely, the statistical properties of $\boldsymbol{u}$ are specified).

(2) Linear in $\boldsymbol{B}$: the nonlinearities of the dynamo problem arise in three ways—first is the nonlinear coupling of the flow and field inherent in the field-line stretching term of the induction equation (i.e. the term $\nabla \times (\boldsymbol{u} \times \boldsymbol{B})$); second is the interaction of the current and field to produce the Lorentz force in the Navier–Stokes equation (the term $(1/c)\boldsymbol{J} \times \boldsymbol{B}$); third is the nonlinear advection term in the Navier–Stokes equation, which is ultimately responsible for the nonlinear behaviour of the fluid (the term $\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$). However, since $\boldsymbol{u}$ is specified in the kinematic approach, these three nonlinearities do not enter the problem.

(3) Scale-separated: because the initial aim is to account for the large-scale long-time behaviour of the stellar magnetic field, one averages the induction equation (3.4) over spatial/temporal scales larger than the characteristic scales of the turbulent flow in the convection zone of the star.

Thus, one writes in the simplest case (Roberts 1994)

$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \boldsymbol{b}; \qquad \boldsymbol{u} = \delta \boldsymbol{u}, \tag{3.8}$$

where $\langle \boldsymbol{B} \rangle$ is the averaged field, $\boldsymbol{b}$ is the small-scale field and we have assumed that the mean flow ($\langle \boldsymbol{u} \rangle$) vanishes. One can then formally write down the evolution equation for the small-scale fluctuating field,

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right)\boldsymbol{b} = \nabla \times \delta\boldsymbol{u} \times \langle \boldsymbol{B} \rangle + \nabla \times (\delta\boldsymbol{u} \times \boldsymbol{b} - \langle \delta\boldsymbol{u} \times \boldsymbol{b} \rangle), \tag{3.9}$$

whose formal solution (obtained after the first-order smoothing assumption $\delta\boldsymbol{u} \times \boldsymbol{b} - \langle \delta\boldsymbol{u} \times \boldsymbol{b} \rangle \sim 0$ is applied) is then inserted into the evolution equation for the mean field.

The consequence of these manipulations (including assuming a specific form for the velocity correlation function) is a mean-field equation for the large-scale field, of the form

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right)\langle \boldsymbol{B} \rangle = \alpha \nabla \times \langle \boldsymbol{B} \rangle, \quad \alpha \sim \langle \delta u \cdot \nabla \times \delta u \rangle, \tag{3.10}$$

which mathematically describes the so-called 'α-effect'. (The α-effect is related to the fact that in a flow for which the fluid helicity (α) does not vanish, small-scale fluid motions will twist and stretch the local field so that the current associated with these field perturbations flows along the local mean-field direction; that is the reason why the source term on the right-hand side of the mean-field induction equation (3.10) is proportional to the local current density.) A more careful treatment shows that one also obtains (to the same order) a term proportional to $\nabla^2\langle \boldsymbol{B}\rangle$, i.e. an additional diffusion term (the 'turbulent diffusion' term); and one can remove the restriction that the mean flow should vanish; thus, one obtains the full mean-field equation

$$\left(\frac{\partial}{\partial t} + \langle \boldsymbol{u}\rangle \cdot \nabla\right)\langle \boldsymbol{B}\rangle = \alpha\nabla \times \langle \boldsymbol{B}\rangle + \eta_t \nabla^2 \langle \boldsymbol{B}\rangle. \tag{3.11}$$

One now specifies the functionals α and η, as well as $\langle \boldsymbol{u}\rangle$, as explicit functions of position, and solve equation (3.11); in the solar context, the mean flow $\langle \boldsymbol{u}\rangle$ is typically taken as the shear flow which leads to differential rotation with radius in the solar interior (the 'Ω-effect'). This type of dynamo model—the 'α–Ω' dynamo—was first developed (Parker 1955) in the context of trying to account for the basics of solar magnetic-field observations—the 'Butterfly diagram', which describes the temporal variations in magnetic-flux emergence during the course of the 22-year magnetic solar cycle; Hale's polarity law, which describes the north–south and east–west orientation of large emerging bipolar field complexes; and the fundamental cycle period duration of *ca.* 22 years. It is a remarkable tour-de-force that even with fairly naive, but physically plausible, evaluations of α and η_t, this model is able to reproduce in the main the large-scale long-duration behaviour of the Sun's magnetic field, the large-scale spatial patterns of flux emergence and the cycle period.

Given this success, why go further? There are a number of very good reasons for persevering.

(1) The theory assumes that the field fluctuations are much smaller than the mean field; observations tell us that precisely the opposite is true.

(2) The theory assumes that scale separation makes sense; observations tell us that the field exists on all scales, ranging from scales much smaller than the integral scale of the turbulent fluid, to scales comparable with the size of the Sun.

(3) The solar magnetic field does not grow exponentially with time, unlike the solutions to the mean-field equation—the solar field is presumably in a nonlinearly saturated state, whereas the formalism of mean-field theory regards the field as an exponentially growing quantity. Thus, it seems dubious in the extreme that a linear model could describe in any detail a system which by its very nature must function in a nonlinear regime.

(*d*) *The 'solar–stellar connection' scenario*

If we for the moment suppress our skepticism, and adopt the mean-field model, it is then possible to outline a 'scenario'† which has some hope of correctly describing the physics of stellar magnetic activity. The chain of reasoning goes as follows.

† A 'scenario' in this context refers to a verbal description of a sequence of physical processes; typically, limited aspects of this sequence, as well as limited numbers of physical processes in this sequence, may be mathematically or computationally approachable calculations, but the entire sequence is usually completely inaccessible to realistic computations.

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(1) We assume that an 'α–Ω' dynamo functions in the solar interior. The rate of flux production is presumed to depend on the stellar rotation rate. (In fact, since the actual solar dynamo is nonlinearly limited, the flux production rate cannot simply, in isolation, depend on the rotation rate alone; linear theory is, however, incapable of dealing with this key point.)

(2) The magnetic flux produced by this interior dynamo rises to the surface, where its interactions with the star's surface motions lead to dissipative effects in the overlying atmosphere; this dissipation produces the star's hot corona (Parker 1979).

(3) The continual flux emergence, together with the heating of the outer atmosphere, lead to the generation of a magnetized wind outflow (Belcher & Olbert 1975; Belcher & MacGregor 1976).

(4) This magnetized wind acts to despin the star (Kraft 1967; Weber & Davis 1967; Belcher & Davis 1971; Soderblom *et al*. 1993), so that the stellar rotation rate decreases. This happens on a stellar evolution time-scale, and so much longer than the magnetic cycle period.

(5) This gradual decrease in stellar rotation rate leads to a related gradual decrease in the stellar activity level, as measured by the various emission processes.

(6) The decreased activity level leads to a less vigorous magnetized wind, and whence to less effective stellar spin-down.

No one has been able to—nor has even attempted—to solve this full problem directly, but recently there have been attempts to formulate and solve parts of this 'scenario'; for example, MacGregor & Charbonneau (1997; see also Charbonneau & MacGregor 1997) have in a recent series of papers tackled items (3) and (4), including modelling the change in interior rotation—the differential rotation. In contrast, items (1) and (2), as well as (5) and (6), remain out of reach of direct numerical simulations.

(*e*) *The role of numerical computations*

In the above discussion, I have been (intentionally) somewhat imprecise whenever talking about calculations. In principle, of course, what we would ideally like to obtain are numerical solutions of the full set of MHD equations (3.4), (3.5). This ideal is practically unattainable because the spatial (and temporal) dynamic range of the real physical system is many orders of magnitude larger than what is achievable on even the largest imaginable supercomputer (much less on existing machines). We are thus left with a few, less desirable, choices; and we need our imagination to make the best of this.

(1) We can carry out direct numerical simulations (DNS) of very small versions of the 'real system'. Such calculations have the virtue that there are no fitting parameters, and no models (of, for example, the subgrid dynamics). In the present context, such calculations have been enormously successful in helping our understanding of fast dynamos, and their relationship to the possible highly localized flux generation near the surface of a star like the Sun (see, for example, Cattaneo *et al*. 1996; Tao

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et al. 1998); thus, one can make a very strong case, on the basis of such DNS calculations, that much of the small-scale magnetic flux on the solar surface— the 'salt and pepper' commonly seen in solar magnetograms—is most likely generated in the top layers of the solar convection zone.†

(2) We can construct lower-dimensional nonlinear model systems, and use them to guide our intuition regarding new effects introduced by nonlinearities. Such dimension reduction is sometimes carried out by truncating the full nonlinear equations of motion (Weiss *et al*. 1984). More reliable results are obtained by applying the method of normal forms (Arnold 1983), which has the significant advantage that it can be used to construct the phase portrait for the behaviour of solutions to the fully nonlinear system of differential equations; this method has been applied to the dynamo problem (Tobias *et al*. 1995; Knobloch *et al*. 1998). These types of calculation have been successful in teaching us, for example, how the very-low-activity periods known generically as 'Maunder minima' can come about (Weiss 1996).

(3) We can construct stellar dynamo calculations in the spirit of 'global circulation' models familiar to fluid dynamicists in climate research, i.e. we construct models that describe the physics occurring on unresolved spatial scales. Such calculations, especially in MHD, are still in their infancy, and require substantially more effort in constructing reasonable models for the small-scale dynamics. An intriguing path for progress may be collaborations with laboratory plasma groups, in which models for subgrid dynamics are tested against laboratory experiments; examples which are being looked at in some detail in Chicago include validation calculations of laboratory Kruskal–Schwarzschild instability and of reconnection experiments at modest values of the Lundquist number ($\equiv v_{\mathrm{A}}L/\eta = (v_{\mathrm{A}}/u)Re_{\mathrm{m}}$, where u and v_{A} are the RMS fluid velocity and the Alfvén velocity, respectively, and Re_{m} is the magnetic Reynolds number).

4. The major puzzles

As my final step, I would like to summarize what I regard as the outstanding problems in this subject area.

(*a*) *Why do simple models well describe the large-scale behaviour, or How does the large-scale solar dynamo work?*

The puzzle is as follows: as discussed, we have considerable evidence that the classical 'α–Ω' dynamos capture much of the behaviour of the observed low-order multipoles of the solar magnetic activity cycle; thus, these relatively simple models give a good account of the temporal behaviour of the solar cycle (namely, the cycle period), as well as of the spatial morphology (including the butterfly diagram of sunspot eruption latitude as a function of time, and the Hale polarity law). One way to interpret this agreement between the observations and the linear model is that the dominant nonlinear eigenfunctions for the actual Sun resemble at small wavenumbers (large scales) the linear eigenfunctions of linear kinematic ('α–Ω') dynamo theory.

† This result raises an interesting observational question: what is the partitioning of stellar magnetic flux between 'spots' and small-scale fields? If one could answer this question, then one might be able to separately constrain the (slow) mechanism responsible for the large-scale spot-related fields and the (fast) mechanism responsible for the 'salt and pepper' flux elements.

Nevertheless, we know that the solar dynamo operates in a fully nonlinear state, whereas the linear kinematic models do not by definition. How can this be?

One possible solution is that the Sun's large-scale (nonlinear) dynamo is marginal, in the sense that it actually operates close to the weakly nonlinear regime. This suggestion is consistent with the success of low-order (truncated) nonlinear models of the solar dynamo (Jones *et al*. 1985; Weiss 1994, and references therein) in accounting for the long-term modulation of the solar cycle (including occasional 'Maunder minima').

(*b*) *Dynamo and activity 'saturation'*

Observations of activity indicators for the most active stars have shown that the power-law scaling of activity-related emission with (for example) the stellar rotation rate disappears—the activity process is said to 'saturate' (Mathioudakis 1999; Stern 1999). Not surprisingly, a number of possible explanations have been put forward, a number of which are well described by the following quote from Solanki *et al*. (1997):

> ... we expect stellar dynamos to eventually saturate at sufficiently high rotation rates, since at some point the back-reaction of the magnetic field on convection and differential rotation becomes appreciable.

Is the physical picture suggested by Solanki *et al*. (1997) sensible? To answer this question, I first recall that in classical mean-field dynamo theory, the evolution equation for the mean magnetic field is, first, kinematic, and, second, linear in the mean field; the relevant (interesting) solutions grow exponentially in time. However, no observed 'natural' dynamos have 'solutions' that are exponential in time—that is, all observed magnetic fields in nature show 'saturation', in the sense that their fluctuation amplitudes are bounded as a function of time. This type of saturation must arise from nonlinear interactions, i.e. ones that take into account the backreaction of the magnetic field on the fluid motions. This, however, is *not* the type of 'saturation' that Solanki *et al*. (1997) are referring to; and furthermore it is not easy to see why one would naturally expect the Lorentz force backreaction to 'saturate' the emission rotation-rate power-law scaling.

This does not mean, of course, that one cannot come up with a number of candidate mechanisms that could account for the observed changes in activity behaviour at large rotation rates; examples include the following.

(1) Changes in the linear mode structure at large rotation rates that survive into the nonlinear regime; an example would be a transition (for the linear problem) from an 'α–Ω'-type dynamo to an 'α^2' or 'α^2–Ω'-type dynamo.

(2) Changes in action of the nonlinearities that lead to saturated solutions at any fixed rotation rate when rotation rates become particularly large.

(3) Introduction of new nonlinearities at large rotation rates. These nonlinearities can enter because stellar dynamos do not exist in isolation. Their output (magnetic fields) regulates the heating of stellar coronae and the rate of mass outflow via winds, while the wind losses feed back (negatively) to the stellar rotation rate. Thus, we really need to solve the coupled system consisting of: (a) the stellar interior convection zone; (b) the solar dynamo; and (c) the

outer atmosphere, including the corona and the out-flowing wind. In that case, loss of scaling could result from (for example) enhanced stellar spin-down (as a result of increased surface activity levels), and consequent changes in the global interior circulation.

(4) 'Displacement' of the emission region. Solar observations show that X-ray and similar activity-related emission in solar active regions comes primarily from regions *surrounding* sunspots, not from the sunspots themselves (Vaiana & Rosner 1978). Thus, if starspots take up more and more of a star's surface as the rotation rate increases, the fraction of the stellar surface from which this emission derives can actually decrease.

(*c*) *What happens at the end of the main sequence?*

As the final puzzle, I offer the fate of activity at the low-mass end of the main sequence. We know from surveys of the nearby stars (Schmitt *et al*. 1995; Schmitt 1997) that stellar activity as measured by the X-ray-to-bolometric luminosity ratio does not change significantly as one proceeds down the main sequence to low-mass stars. However, surveys of very low mass stars (e.g. stars that may be, or are, classified as brown dwarfs) suggest that some of them may well be inactive (Schneider *et al*. 1991; Basri & Marcy 1995; Linsky *et al*. 1995; Basri *et al*. 1996; Neuhäuser *et al*. 1999). Although the already mentioned caveat—lack of evidence is not evidence for absence—must be kept in mind, we in fact know that some sort of transition from convection-driven activity must occur at sufficiently low mass: For example, we know that the observed activity of the giant gaseous planets in our solar system is not solar-like at all, but is instead the result of the interaction of planetary magnetospheres (whose plasma loading is dominated by their moons) with the solar wind; for gaseous planets, magnetic-field 'stirring' at the planetary surfaces by surface flows is a negligible source for their activity, while such stirring by photospheric motions is precisely how we believe stellar chromospheres and coronae are energized.

Thus, we already have direct evidence that rotating self-gravitating objects (stars and giant gaseous planets) produce magnetic fields in their interior via a dynamo process: that these objects are all 'active'; and that the underlying basic physics that drives the activity in these two cases is drastically different in its origins. It may be of some interest that some 20 years or so ago, Arons (1979) suggested that stellar objects which were magnetized, but which did not energize their magnetospheres via surface convective motions, might nevertheless be 'active' as long as a mechanism existed that could plasma load the magnetospheres, and as long as the object was in an environment that could perturb the plasma-loaded magnetosphere. What remains unclear is exactly how the transition from 'coronal' to 'magnetospheric' behaviour occurs; one likely factor ought to be the fractional ionization at the object's surface, as well as the microscopic transport properties at the surface.

A related issue revolves around the nature of convection, and the nature of the interior dynamo, as one proceeds to lower and lower-mass objects. Classical stellar-structure theory predicts that stars become fully convective at spectral type $\sim$ dM5 (Clayton 1983), and based on recent ideas that the base of the convection zone should play a particularly important role in magnetic dynamos (Spiegel & Weiss 1980), one would have expected (and did expect; Rosner 1980) to see a transition in activity behaviour at $\sim$ dM5—yet no such transition has been observed. How can we

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understand this? Is it possible that, in the presence of even relatively modest interior magnetic fields, low-mass stars never become fully convective? Could such magnetic fields stabilize the deep interior in the mean? It is evident that much remains to be done, and that it will be fun to do it!

I thank the organizers, as well as The Royal Society and the Novartis Foundation, for the opportunity to participate in this meeting, as well as for funding support. I also thank Professor N. O. Weiss for cogent comments and useful suggestions on the manuscript.

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Discussion

I. W. Roxburgh (*Queen Mary and Westfield College, UK*). I feel you're putting too much weight on the two observations of brown dwarfs.

R. Rosner. I agree that one needs to be cautious, but they are the only ones we have. It could be that the one that doesn't show much alpha activity happens to be in a Maunder-minimum-like state. It's a classic example of a lack of evidence not being evidence of absence.

A. Brandenburg (*University of Newcastle upon Tyne, UK*). You showed a plot about the observations of Baliunas of the cycle period divided by rotation period versus inverse rotation period and you showed that this was an increasing function. We have actually looked at the same data and have particularly focused on the data that have good quality. If you plot the ratio of cycle frequency to rotation frequency versus the Rossby number, then you find a clear correlation but they fall into two different branches, which show exactly the opposite trend. One branch increases, and the second also increases with inverse Rossby number, but it is shifted downwards.

R. Rosner. I'm aware of that. In defence of Sally's work, she chose that plot because it related parameters which are purely observational and therefore determined. The Rossby number depends on a quantity which you don't know—it is model dependent. There may be systematic effects that you introduce because you're dependent on the model. So she was trying to avoid that problem. One question to answer is: what would change if you took her dataset and cleaned it in the way you did, that is only restricted to systems where you're sure of the parameters and re-plotted it? Suppose you clean the dataset just the way you described but instead just repeated her plot; would you find the same result as her or not?

N. O. WEISS (*University of St Andrews, UK*). What does come out clearly is that, at least for stars which have cycles like the Sun, the cycle period goes down once the rotation rate increases.